Equilibrium Spectra of Secondary Cosmic-Ray Positrons in the Galaxy and the Spectrum of Cosmic Gamma Rays Resulting From Their Annihilation

7-1 INTRODUCTION

The annihilation of cosmic-ray positrons has for some time been recognized as a potential source of cosmic γ-rays. Gamma-ray fluxes from cosmic positron annihilation have been estimated and discussed by various authors (Pollack and Fazio, 1963; Hayakawa et al., 1964; and Ginzburg and Syrovatskii, 1964). Pollack and Fazio (1963) have discussed the possible relationship between the present flux of 0.5-MeV γ-radiation from positron annihilation and the cosmic-ray intensity and galactic gas density 10^9 years ago. Ginzburg and Syrovatskii (1964) have pointed out that the intensity of the 0.5-MeV line may be a sensitive measure of the leakage rate of cosmic-ray positrons from the galaxy. These authors have also given an approximate formula for the calculation of the γ-ray spectrum from the annihilation of high-energy positrons.

It has therefore become apparent that the cosmic γ-ray spectrum from cosmic-ray positron annihilation may contain much potential information reflecting various astrophysical conditions in our own galaxy and in possible cosmic-ray sources, both galactic and extragalactic.

Cosmic-ray positrons may be produced in the galaxy either from the decay of secondary π^+-mesons via the decay chain

$$\begin{array}{l} \pi^+ \rightarrow \mu^+ + \nu_\mu \\ \qquad\quad \hookrightarrow e^+ + \nu_e + \bar{\nu}_\mu \end{array} \tag{7-1}$$

As was the case with the neutral pions discussed previously, the positive pions may be produced either in cosmic-ray interactions with the galactic gas or in matter-antimatter annihilations.

In addition to the positrons resulting from pion decay, lower energy positrons are produced in the galaxy, principally by cosmic-ray p—^{12}C, p—^{14}N, and p—^{16}O collisions at relatively low energy, which result in the production of radionuclei having β-ray-positron-emitting decay modes.

Since these interactions are nuclear transmutations rather than pion-producing interactions, they involve relatively low threshold energies (of the order of 10 MeV rather than 300 MeV). Moreover, the β-ray positrons are produced in the galaxy, principally by cosmic-ray p–^{12}C. whereas the spectrum of pion-decay positrons peaks at about 35 MeV and decreases rapidly toward lower energies. Thus, if there exists a large flux of low-energy galactic cosmic rays, these particles may provide the most copious source of low-energy cosmic-ray positrons. This source may then contribute significantly to an observable flux of positrons near the Earth at and below 1-MeV energy. Furthermore, because of their low energy, β-ray positrons are much more likely to stop and annihilate in the galaxy to produce 0.51-MeV γ-rays than are the positrons from the pion-decay process. Reversing the argument leads to the conclusion that the intensity of the 0.51-MeV positron-annihilation line in the galaxy may provide a sensitive measure of the galactic cosmic-ray flux below 1 GeV/nucleon.

A typical cosmic-ray positron may undergo one of three fates: it may escape from the galaxy, annihilate with an electron while at relativistic energy, or lose almost all of its energy before annihilation. One must, of course, consider that annihilations may occur either between free electrons and positrons or through the formation of intermediate bound states of positronium. All of the physical processes listed in this section must be considered in calculating the final positron annihilation γ-ray spectrum.

7–2 THE POSITRON PRODUCTION SPECTRUM FROM THE DECAY OF POSITIVE PIONS

Positron production arising from pion-producing cosmic-ray interactions followed by the decay process (7–1) has been discussed in many places in the literature (Hayakawa, 1963; Hayakawa and Okuda, 1962; Hayakawa et al., 1964; Ginzburg and Syrovatskii, 1964; Jones, 1963, 1965a; Pollack and Fazio, 1965; Scanlon and Milford, 1965; Ramaty and Lingenfelter, 1966a, b, 1968; and Perola, Scarsi, and Sironi, 1968). We present here a brief review of the kinematics of the decay along with the results of the most recent calculations by Ramaty and Lingenfelter.

The decay process (7–1), being a three-stage decay chain, will involve three intermediate integrations of the form of equation (1–14) in order to determine the positron production spectrum. The decay

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \tag{7–2}$$

is a two-particle decay involving one massless particle (the neutrino),

so equation (1–78) is applicable. From equations (1–65) and (1–78), we obtain

$$E_\mu = \frac{E_\pi}{M_\pi}\left[\left(\frac{M_\pi^2 + M_\mu^2}{2M_\pi}\right) + \left(1 - \frac{M_\pi^2}{E_\pi^2}\right)^{1/2}\left(\frac{M_\pi^2 - M_\mu^2}{2M_\pi}\right)\cos\theta'\right] \tag{7–3}$$

or, using η and ρ,

$$E_\mu = E_\pi[\eta(M_\pi, M_\mu) + \beta_\pi\rho(M_\pi, M_\mu)\cos\theta'] \tag{7–4}$$

As was shown in chapter 1 (e.g., see eq. (1–147)), equation (7–4) can also be expressed as

$$E_\pi = \frac{M_\pi^2}{M_\mu^2}[\eta(M_\pi, M_\mu) + \tfrac{1}{2}\beta_\mu\kappa(M_\pi, M_\mu)\cos\theta']E_\mu \tag{7–5}$$

through the use of equation (1–42); equation (7–5) can be used to place limits on the energy range of pions which produce muons of energy E_μ. We thus obtain

$$f(E_\mu) = \int_{E_{\pi,\min}}^{E_{\pi,\max}} dE_\pi \frac{f(E_\pi)}{\kappa(M_\pi, M_\mu)(E_\pi^2 - M_\pi^2)^{1/2}} \tag{7–6}$$

where

$$E_{\pi,\substack{\max\\\min}} = \frac{M_\pi^2}{M_\mu^2}[\eta(M_\pi, M_\mu) \pm \tfrac{1}{2}\beta_\mu\kappa(M_\pi, M_\mu)]E_\mu \tag{7–7}$$

Equation (7–7) may be numerically approximated by

$$E_{\pi,\substack{\max\\\min}} = (1.36 \pm 0.36\beta_\mu)E_\mu \tag{7–8}$$

and, as $\beta_\mu \to 1$,

$$\begin{aligned} E_{\pi,\max} &\to \left(\frac{M_\pi}{M_\mu}\right)^2 E_\mu \simeq 1.72E_\mu \\ E_{\pi,\min} &\to E_\mu \end{aligned} \tag{7–9}$$

The decay

$$\mu^+ \to e^+ + \nu_e + \nu_\mu \tag{7–10}$$

is a three-body decay with a resultant broad energy spectrum of the positron in the rest system of the muon. Let us denote the normalized positron energy spectrum in the muon rest system by $D_\mu(E_e')$. It has been shown that

$$D_\mu(E_e') = \frac{4E_e'}{E^{*4}}(E_e'^2 - M_e^2)^{1/2}\left[3(E^* - E_e') + \frac{2}{3}\omega(4E_e' - 3E^*)\right] \tag{7–11}$$

where E^* is the maximum energy available to the electron in the decay,

$$E^* = \frac{M_\mu}{2}\left[1 + \frac{M_e^2}{M_\mu^2}\right] \simeq \frac{M_\mu}{2} \tag{7-12}$$

and $\omega \simeq 3/4$ is the Michel parameter (Michel, 1949).

From equations (1–42) and (1–49) we obtain the relation between E_μ and E_e' for a given value of E_e as

$$E_\mu = \frac{M_\mu}{M_e^2} E_e E_e'(1 + \beta_e \beta_e' \cos\theta') \tag{7-13}$$

which sets the limits on the energy range of muons which produce positrons of energy E_e.

Putting all of these results together in a manner similar to that used in chapter 1 for obtaining formulas for γ-ray production spectra, we obtain

$$q(E_{e^+}) = 2\pi n \int_{E_{\mathrm{th}_p}}^{\infty} dE_p I(E_p) \int_{M_e}^{E^*} \frac{dE_e' D(E_e')}{\sqrt{E_e'^2 - M_e^2}} \int_{(M_\mu/M_\pi^2)E_e E_e'(1-\beta_e\beta_e')}^{(M_\mu/M_\pi^2)E_e E_e'(1+\beta_e\beta_e')} dE_\mu$$
$$\times \int_{(M_\pi^2/M_\mu^2)(\eta-\beta_\mu\rho)E_\mu}^{(M_\pi^2/M_\mu^2)(\eta+\beta_\mu\rho)E_\mu} dE_\pi \frac{\sigma(E_\pi|E_p)}{(E_\pi^2 - M_\pi^2)^{1/2}} \tag{7-14}$$

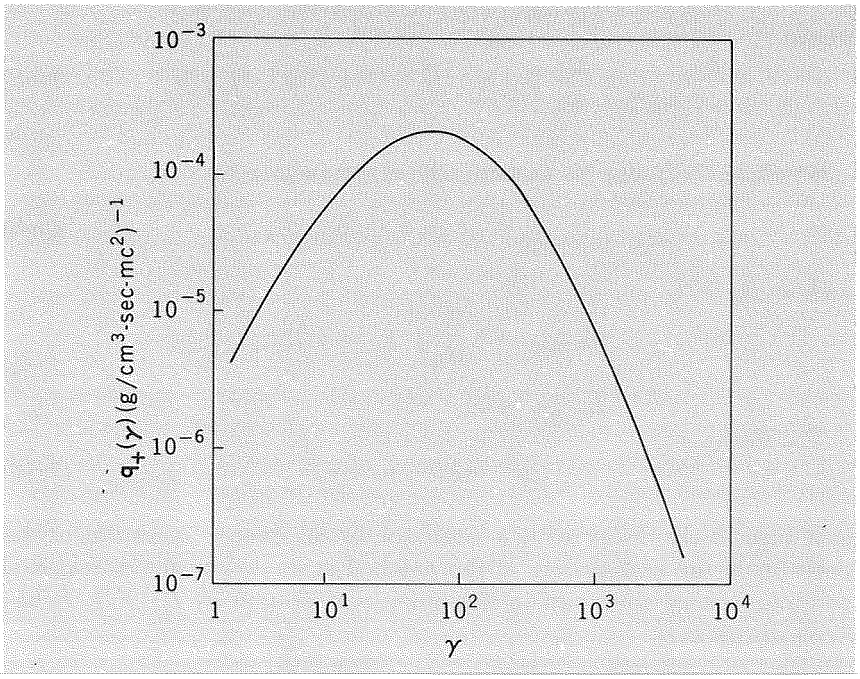

FIGURE 7–1.—The source spectrum of positrons produced from the decay of positive pions formed in cosmic-ray collisions.

It is of interest to note that most calculations of the positron production spectrum from pion decay have included the approximation of neglecting the muon recoil in the pion decay by setting $\beta_\mu \simeq \beta_\pi$. In this approximation, the right-hand integral in equation (7–14) is eliminated, and the muon and pion production functions obey the relation

$$\sigma_\mu(E_\mu|E_p) = \frac{M_\pi}{M_\mu}\,\sigma_\pi\left(\frac{M_\pi}{M_\mu}E_\mu\middle|E_p\right) \tag{7–15}$$

Ramaty and Lingenfelter (1966a) have shown that this approximation introduces little error into the calculation.

The most recent determination of the positron production spectrum from charged pion decay, as calculated by Ramaty, is shown in figure 7–1.

7–3 THE POSITRON PRODUCTION SPECTRUM FROM THE DECAY OF RADIONUCLEI

The production spectrum of radionuclei $q_r(E_r)$ is given by (Ramaty, Stecker, and Misra, 1970)

$$q_r(E_r) = 4\pi \sum_{i,k} \int dE_i\, I_i(E_i) n_k \sigma_{r,ik}(E_i) f_{r,ik}(E_r|E_i) \tag{7–16}$$

where E_i and E_r are the energies of the primary and secondary particles, respectively; I_i is the intensity of primary cosmic-ray nuclei of type i in interstellar space; n_k is the number density of interstellar gas atoms of type k; $\sigma_{r,ik}$ is the interaction cross section for production of a radionucleus of type r, and $f_{r,ik}(E_r|E_i)$ is the normalized energy distribution function for production of a nucleus with energy E_r in an interaction of energy E_i.

Among the variety of β-emitting nuclei which may be formed in galactic cosmic-ray interactions, only ^{11}C, ^{13}N, ^{14}O, and ^{15}O contribute significantly to positron production. The principal reactions and their threshold energies are given in table 7–1 along with their decay modes and maximum positron energies. The cross sections were summarized by Lingenfelter and Ramaty (1967) and by Audouze, Epherre, and Reeves (1967). The distribution functions $f_{r,ik}(E_r|E_i)$ are, in general, unknown. Their detailed form, however, is not important for the present problem since most of the positrons are emitted from nuclei decaying near rest. Thus, the positron production spectrum is determined mainly by the β-ray spectrum of the parent nuclei. Ramaty et al. (1970) assumed that in these p–CNO interactions, the kinetic energy per nucleon is equally distributed among all the secondary nuclei produced, so that

$$f_{r,ik}(E_r|E_i) = \delta\left[E_r - A_r\left(\frac{E_i - M_i - Q_{ik}}{A_i + A_k}\right) - M_r\right] \tag{7–17}$$

TABLE 7–1.—*Principal Reactions Leading to Production of β-Emitting Nuclei*

[See Audouze et al., 1967]

Reaction	σ, mb	Decay	Half-life	Maximum positron energy, MeV
$^{12}C(p, 3p2n)^{8}B$	?	$^{8}B(\beta^+)^{8}Be$	0.78 s	1.4
$^{12}C(p, p2n)^{10}C$	~3	$^{10}C(\beta^+)^{10}B$	19 s	1.9
$^{14}N(p, 2p3n)^{10}C$	?			
$^{16}O(p, 3p, 4n)^{10}C$	<10			
$^{12}C(p, pn)^{11}C$	~70	$^{11}C(\beta^+)^{11}B$	20.5 min	0.97
$^{14}N(p, 2p2n)^{11}C$	~30			
$^{16}O(p, 3p3n)^{11}C$	~10			
$^{12}C(p, n)^{12}N$	?	$^{12}N(\beta^+)^{12}C$	0.011 s	16.4
$^{14}N(p, p2n)^{12}N$	?			
$^{16}O(p, 2p3n)^{12}N$	?			
$^{14}N(p, pn)^{13}N$	~15	$^{13}N(\beta^+)^{13}C$	10.0 min	1.19
$^{16}O(p, 2p2n)^{13}N$	~10			
$^{14}N(p, n)^{14}O$	[a]~50	$^{14}O(\beta^+)^{14}N$	71 s	1.18, 4.14
$^{16}O(p, p2n)^{14}O$	<10			
$^{16}O(p, pn)^{15}O$	~50	$^{15}O(\beta^+)^{15}N$	2.06 min	1.74

?=Unknown and not included in estimate of positron production.

[a] This cross section has a value of about 10 mb at energies below 12 MeV but is negligible above 150 MeV.

where A_i and A_K are the mass numbers of the incident and target nuclei, respectively.

They assumed for the purposes of calculation that the ratios H:C:N:O in the interstellar gas are $1 : 3.4 \times 10^{-4} : 1.1 \times 10^{-4} : 5.8 \times 10^{-4}$, based on the composition of the solar atmosphere (Durgaprasad et al., 1968), which agree reasonably well with the universal abundances given by Suess and Urey (1956) and Cameron (1959).

The cosmic-ray energy spectrum and charge composition at solar minimum has been measured by a number of experimenters. The proton spectrum above 20 MeV was summarized by Gloeckler and Jokipii (1967), and the spectrum below 20 MeV is given by Fan et al. (1968). The CNO spectra above 50 MeV/nucleon were summarized by Meyer (1969). Because of the lack of data at lower energies, we have assumed that these spectra are of the form $E_i^{0.4}$, normalized to observations at higher energies.

The interstellar primary cosmic-ray intensities can be obtained from the solar minimum spectra through multiplication by a modulation function of the form

$$\mathscr{M}(R) = \begin{cases} \exp\ (R_r/R_0\beta) & \text{for } R < R_0 \\ \exp\ (R_r/R\beta) & \text{for } R > R_0 \end{cases} \tag{7-18}$$

where R and β are rigidity and velocity respectively, R_0 is a characteristic transition rigidity that depends on the distribution of interplanetary magnetic field irregularities, and R_r is a parameter which is space and time dependent and defines the total residual modulation. This modulation function was recently discussed by Ramaty and Lingenfelter (1969). They have shown that above a rigidity of 500 MV the observed cosmic-ray ^{2}H and ^{3}He spectra are consistent with those obtained from nuclear interactions in interstellar space, with $R_r \approx 350$ MV and a mean pathlength for galactic cosmic rays of $X \approx 4$ g/cm^2. Since there is no information on residual modulation at lower energies, we have to treat R_0 as a free parameter.

Since the lifetimes of the CNO β-emitters are short, the positron production spectrum q_+ can be obtained directly from the spectra q_r by taking into account the kinematics of the decay. Thus,

$$q_+(E_+) = \sum_r \frac{1}{2} \int_{M_e}^{E^*} \frac{dE'_+ P(E'_+)}{(E'^2_+ - M_e^2)^{1/2}} \int_{E_{r,\,\min}}^{E_{r,\,\max}} \frac{dE_r\, q_r(E_r)}{(E_r^2 - M_r^2)^{1/2}} \tag{7-19}$$

where E_r, E'_+, and E_+ are the total energies of the decaying isotope, the positron in the rest frame of the isotope, and the positron in the galactic rest frame, respectively; E^* is the maximum value of E'_+ and is given in table 7–1 for the various isotopes involved. The function $P(E'_+)$ is the normalized distribution function for positron production from beta-decay processes and is given by Fermi (1934) as

$$P(E'_+) \propto E'_+ (E'^2_+ - M_e^2)^{1/2} (E^* - E'_+)^2 \tag{7-20}$$

The limits $E_{r\,\min}^{\max}$ are found once again by using equations (1–42) and (1–49) and are given by

$$E_{r,\,\min}^{\max} = \frac{M_r}{M_e^2} E_+ E'_+ (1 \pm \beta_+ \beta'_+) \tag{7-21}$$

Using the data previously discussed, Ramaty et al. (1970) have numerically solved equations (7–16) and (7–19) for the solar minimum spectrum and for $R_r = 350$ MV with various values of R_0. As an extreme assump-

tion, they also used a power-law cosmic-ray spectrum of the form $T_i^{-2.5}$, with low-energy cutoffs T_c at 5, 15, and 100 MeV/nucleon.

The resultant production spectra together with the positrons that would result from pion decay are shown in figure 7–4. As can be seen, the CNO source becomes significant only if there are large fluxes of low-energy cosmic rays. The large positron production at low energies for $R_0 = 200$ MV and for the power-law spectrum comes primarily from ^{13}N and ^{14}O produced by low-energy protons via the reactions $^{16}O(p, 2p2n)^{13}N$ and $^{14}N(p, n)^{14}O$.

7-4 THE EQUILIBRIUM SPECTRUM OF SECONDARY GALACTIC POSITRONS FROM PION DECAY

In order to calculate plausible annihilation γ-ray spectra from our own galaxy, we will assume that the only source of galactic cosmic-ray positrons above a few MeV energy is the result of primary cosmic rays colliding with atoms of interstellar hydrogen and helium gas in the galaxy and producing positive pions which rapidly decay into positrons and neutrinos.

We further assume that these positrons have reached a quasi-equilibrium condition in the galaxy determined by leakage out of the halo, annihilation, and energy loss by ionization and coulomb interactions, bremsstrahlung, Compton collisions, and synchrotron radiation.

The positron equilibrium energy spectrum $n(E_+)dE_+$ then obeys the relation

$$\frac{d}{dt}\int_0^\infty dE_+\, n(E_+, t) = \int_0^\infty dE_+\, q(E_+, t) - \int_0^\infty dE_+\, r_\tau(E_+, t) \qquad (7\text{–}22)$$

where $q(E_+, t)$ is the generating source function of high-energy positrons (in this case, from the decay of mesons formed in high-energy interactions) and $r_\tau(E_+, t)$ is the total positron loss rate due to annihilation and leakage from the galaxy.

In the cosmic medium, energy loss mechanisms such as synchrotron radiation are at work, resulting in a continuous loss of energy per positron. We can take this effect into account by transforming equation (7–22) into

$$\int_0^\infty dE_+ \left[\frac{\partial}{\partial t} n(E_+, t) + \frac{\partial}{\partial E_+}\left(n(E_+, t)\frac{dE_+}{dt}\right)\right] = \int_0^\infty dE_+ \,[q(E_+, t) - r_\tau(E_+, t)] \qquad (7\text{–}23)$$

For convenience, we will now drop the subscript "+" and rewrite equation (7–23) in the differential form

$$\frac{\partial}{\partial t} n(E, t) + \frac{\partial}{\partial E}\,[n(E)r(E)] = q(E, t) - \frac{n(E, t)}{\tau_s(E, t)} \qquad (7\text{–}24)$$

with the loss term $r(E, t)$ rewritten in terms of a survival time $\tau_s(E, t)$ such that for an annihilation time τ_A and a leakage time τ_l,

$$\tau_s(E, t) = \frac{\tau_A(E, t)\tau_l(E, t)}{\tau_A(E, t) + \tau_l(E, t)} \tag{7-25}$$

and

$$r(E) \equiv \frac{dE}{dt} \tag{7-26}$$

The Green's function form of equation (7–24) is

$$\frac{\partial}{\partial t} G[E, t|E', t'] + \frac{\partial}{\partial E} [r(E, t)G[E, t|E', t']] + \frac{G[E, t|E', t']}{\tau_s(E, t)} = \delta(E - E')\delta(t - t') \tag{7-27}$$

In the case where $r(E|t) = r(E)$ and $n_{e-}(t) = n_{e-}$ independent of time, the Green's function for equation (7–27) has the structure

$$G[E, t|E', t'] = \frac{1}{|r(E)|} \exp\left[-\int_E^{E'} \frac{dE''}{\tau(E'')|r(E'')|}\right] \delta\left(t - t' - \int_E^{E'} \frac{dE''}{|r(E'')|}\right) \tag{7-28}$$

Thus, the solution for $n(E, t)$ is

$$n(E, t) = \frac{1}{|r(E)|} \int_E^{\infty} dE'\, q\left(E', t - \int_E^{E'} \frac{dE''}{|r(E'')|}\right) \exp\left[-\int_E^{E'} \frac{dE''}{\tau(E'')|r(E'')|}\right] \tag{7-29}$$

At this point, we will refer back to the basic formulas derived in chapter 2 for calculating the γ-ray spectra from positron annihilation. We will again find it useful to discuss these calculations in terms of the positron Lorentz factor, $\gamma = E/M_e$. Assuming a time-independent source $q_+(\gamma)$, we can then write equation (7–29) as

$$n(\gamma) = \frac{1}{|r(\gamma)|} \int_{\gamma}^{\infty} d\gamma'\, q_+(\gamma') \exp\left[-\int_{\gamma}^{\gamma} \frac{d\gamma''}{|r(\gamma'')|\tau_s(\gamma'')|}\right] \tag{7-30}$$

Equation (7–30) may then be used to obtain numerical solutions for $n(\gamma)$ and the positron equilibrium flux

$$I_+(\gamma) = \frac{c}{4\pi} \frac{n(\gamma)(\gamma^2 - 1)^{1/2}}{\gamma} \tag{7-31}$$

The survival time used will be given by

$$\frac{1}{\tau_s(\gamma)}=\frac{1}{\tau_A(\gamma)}+\frac{1}{\tau_l(\gamma)} \tag{7-32}$$

with $\tau_A(\gamma)$ and $\tau_l(\gamma)$ being the annihilation time and mean leakage time (or diffusion time) for positrons of energy γMc^2. From equation (2–22), we obtain

$$\frac{1}{\tau_A(\gamma)}=\frac{n_e\sigma_0 c}{\gamma}\left(\frac{\gamma-1}{\gamma+1}\right)^{1/2}\left\{\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left[\gamma+(\gamma^2-1)^{1/2}\right]-\frac{\gamma+3}{(\gamma^2-1)^{1/2}}\right\} \tag{7-33}$$

The leakage time will be assumed to be inversely proportional to velocity so that

$$\frac{1}{\tau_l(\gamma)}=\frac{(\gamma^2-1)^{1/2}}{\gamma T_l} \qquad T_l=\text{constant} \tag{7-34}$$

Most of the interstellar gas in our galaxy is un-ionized with the exception of the so-called H-II regions near the very hot O and B stars, which are powerful sources of ionizing ultraviolet radiation. Allen (1963) gives the proportion of space near the galactic plane occupied by clouds of interstellar gas and dust as 7 percent and that occupied by ionized clouds (H-II regions) as 0.4 percent. We will therefore assume that the galactic gas is entirely neutral for the purposes of these calculations and take for the energy loss rate from ionization the expression (see Heitler, 1960; Morrison, 1961):

$$r_I(\gamma)=\frac{8}{3}\sigma_0 n_e c\,\frac{\gamma}{(\gamma^2-1)^{1/2}}\left\{22+\ln\left[\gamma(\gamma-1)(\gamma^2-1)\right]-1.695\left(\frac{\gamma^2-1}{\gamma^2}\right)-\frac{1.39}{\gamma}\right\} \tag{7-35}$$

The energy loss rate from bremsstrahlung may be taken as

$$r_B(\gamma)=7.3\times10^{-16}n_e\gamma \tag{7-36}$$

based on radiation lengths for hydrogen and helium given by Dovzhenko and Pomanskii (1964).

The loss rate from synchrotron radiation and Compton collisions may be taken as

$$r_{s+c}(\gamma)=(1.3\times10^{-9}H^2+3\times10^{-11}\rho_\gamma)\gamma^2 \tag{7-37}$$

where H is given in gauss and the radiation density ρ_γ is given in eV/cm^3 (Ramaty and Lingenfelter, 1966a).

The total positron energy-loss rate is taken to be the sum of equations (7–35) through (7–37), so that

$$r(\gamma) = r_I(\gamma) + r_B(\gamma) + r_{s+c}(\gamma) \tag{7–38}$$

We have now specified all the quantities needed for the numerical solution of equation (7–30). The solution to equation (7–30) then yields the function $n(\gamma)$, which can be used in equation (2–19) to obtain a numerical solution for the in-flight annihilation γ-ray spectrum (AGS) discussed in chapter 2. From equations (2–19) and (7–30), we obtain

$$Q(\eta) = 4H_+ \left(\frac{1}{2}\right) n_e{}^- \sigma_0 c \int_{G(\eta)}^{\infty} d\gamma \frac{\phi(\chi_0(\eta, \gamma), \gamma)}{\gamma(\gamma^2-1)^{1/2}|r(\gamma)|} \times \int_{\gamma}^{\infty} d\gamma' q_+(\gamma') \exp\left[-\int_{\gamma}^{\gamma'} \frac{d\gamma''}{|r(\gamma'')|\tau_s(\gamma'')}\right] \tag{7–39}$$

An interesting case to note in calculating the equilibrium positron spectrum $I_+(\gamma)$ is that for which the source $q_+(\gamma)$ is a source of low-energy positrons, as is the case with the positrons arising from β-decay as discussed in section 7–3. For those positrons, ionization losses predominate so that $r(\gamma) \simeq r_I(\gamma)$, which is proportional to the gas density. We may then reformulate equation (7–30) in a slightly different, but instructive, way.

The leakage lifetime τ_L can be expressed in terms of the mean amount of matter traversed by cosmic rays, $X = \rho c \beta \tau_L$. From cosmic-ray spallation studies, it is found that $X \approx 4 \text{g/cm}^2$.

Equation (7–31) for the equilibrium positron flux $I_+ = n_+ \beta c/4\pi$ can be written as

$$I_+(\gamma) = \frac{1}{4\pi} \frac{1}{K(\gamma)} \int_{\gamma}^{\infty} d\gamma' q_+(\gamma') \exp\left[-\frac{1}{X}\int_{\gamma}^{\gamma'} \frac{d\gamma''}{K(\gamma'')} \frac{1}{1 + X/X_A(\gamma'')}\right] \tag{7–40}$$

where $K(\gamma) = r(\gamma)/\rho\beta c$ (in cm^2/g) is the stopping power of protons in hydrogen, and X_A is an equivalent annihilation pathlength given by

$$X_A(\gamma) = 9.3(\gamma+1)\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1} \ln(\gamma + \sqrt{\gamma^2-1}) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right]^{-1} \quad \text{g/cm}^2 \tag{7–41}$$

As can be seen, the positron intensity I_+ depends only on the interstellar cosmic-ray intensity (through the production function q_+) and on the pathlength X. Furthermore, at low positron energies, where the range of these particles is much smaller than X, I_+ depends only on the cosmic-ray intensity itself.

TABLE 7–2.—*Average Astrophysical Parameters for the Galactic Disk and Halo*

	n_e, cm^{-3}	H, μG	ρ_γ, eV/cm^3	T_l, 10^6 yr	L_{eff}, cm
Disk	0.5–1	6	0.45	2–4	3×10^{22}
Halo	$1–3\times10^{-2}$	3	0.65	100–300	5×10^{22}

In order to utilize equations (7–30) and (7–39) to determine the positron equilibrium flux and AGS in the galaxy, Stecker (1969a) chose typical average values for the quantities n_e, H, ρ_γ, and T_l for both the galactic disk and the galactic halo models. These values are given in table 7–2. The radiation density ρ_γ includes the contribution of 0.25 eV/cm^3 from the 2.7 K universal microwave field (Stokes, Partridge, and Wilkinson, 1967).

Using the values given in table 7–2 for the galactic halo model to integrate equations (7–30) and (7–39) numerically, Stecker (1969a) obtained typical galactic spectra from pion-decay positions for $I_+(\gamma)$ and $Q(\eta)$. The results are given in figures 7–2 and 7–3 for various values of mean pathlength X (g/cm^2) and corresponding T_l.[1] The flux of annihilation γ-radiation observed at the Earth, $I_A(\eta)$, is given by

$$I_A(\eta)=\frac{L_{\text{eff}}}{4\pi}Q(\eta) \tag{7–42}$$

where L_{eff} is the effective pathlength for γ-ray production.

For leakage times less than 10 million years, the equilibrium positron flux has roughly the same characteristics as the source spectrum of figure 7–1, and its magnitude is proportional to the leakage time. In the case of longer leakage times, the positrons are trapped in the galaxy for

[1] The relation between mean pathlength X and mean leakage time T_l is given by the equation $X=\rho cT_l$, where ρ is the mean density of the gas in the medium (Ramaty and Lingenfelter, 1966a). Assuming the galactic medium is 90 percent hydrogen and 10 percent helium, with X in g/cm^2, n_H in atoms/cm^3, and T_l in millions of years, this relation becomes $X\simeq 5n_HT_l$. Thus, for a halo model with $X=5$ g/cm^2 and $n_H=10^{-2}$ cm^{-3}, $T_l=10^8$ years; whereas for a disk model with $X=5$ g/cm^2 and $n_H=1$ cm^{-3}, $T_l=10^6$ years. We choose here to discuss the background γ-ray spectrum from a halo model of the galaxy, since it has been shown by Ramaty and Lingenfelter (1966b) that such a model gives a positron equilibrium spectrum, even in the disk, which is almost identical with that obtained for a disk-plus-halo model. The inclusion of a spatial diffusion term of the form $D_0\nabla^2N$ in equation (7–24) will have little effect for positrons with $\gamma<2\times10^4$. It may also be noted that a recent study by Anand, Daniel, and Stephens (1968) indicates that there is no large gradient of the cosmic-ray electron intensity between the disk and the halo for energies less than 10 GeV.

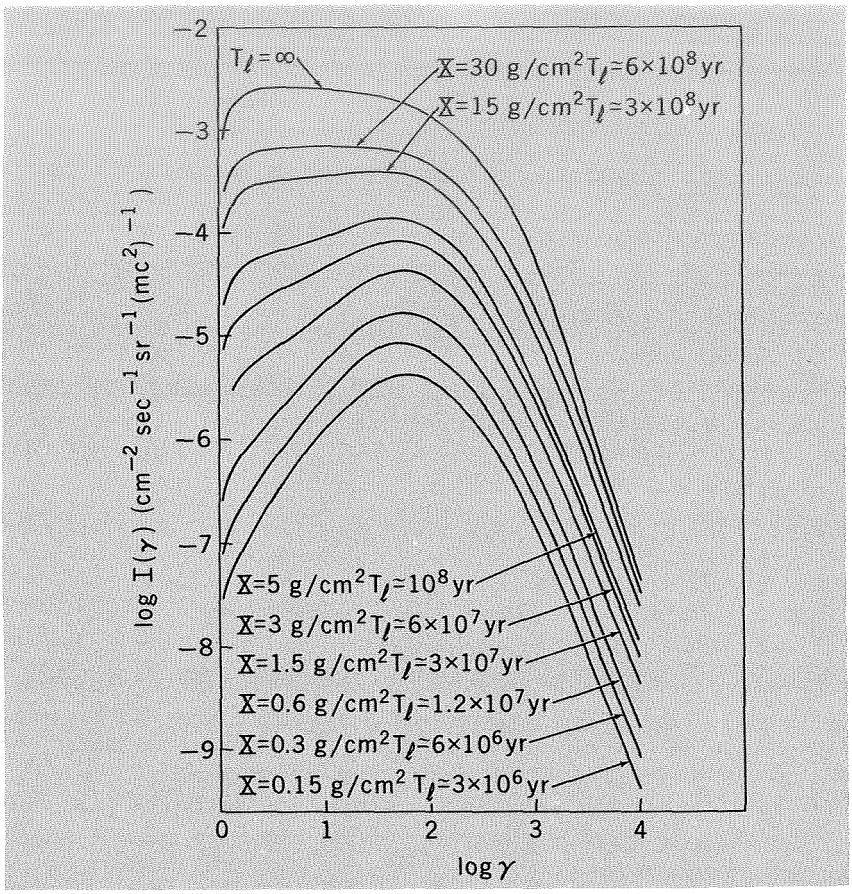

FIGURE 7–2. — Various positron equilibrium fluxes for the halo model of the galaxy given for various approximate mean pathlengths and mean leakage times (Stecker, 1969a).

a sufficient time for the energy loss processes, particularly ionization loss, to affect the spectrum by progressively flattening it below 30 MeV. Studies of spallating cosmic-ray nuclei yield a mean pathlength, $X = 4 \pm 1$ g/cm^2 for cosmic-ray nuclei. The curve in figure 7–2 corresponding to $X = 5$ g/cm^2 is in agreement with the cosmic-ray positron measurements of Hartman (1967). It may be noted that measurements of the galactic positron flux below 30 MeV would yield a more sensitive check on the galactic mean leakage time.

Figure 7–3 shows the annihilation γ-ray spectra obtained using the positron equilibrium fluxes of figure 7–2. The spectra shown are from annihilations in flight of positrons having energies greater than 5 keV. The AGS from positrons annihilating with energies below 5 keV will be discussed in the next section. It can be seen that the peaks of these

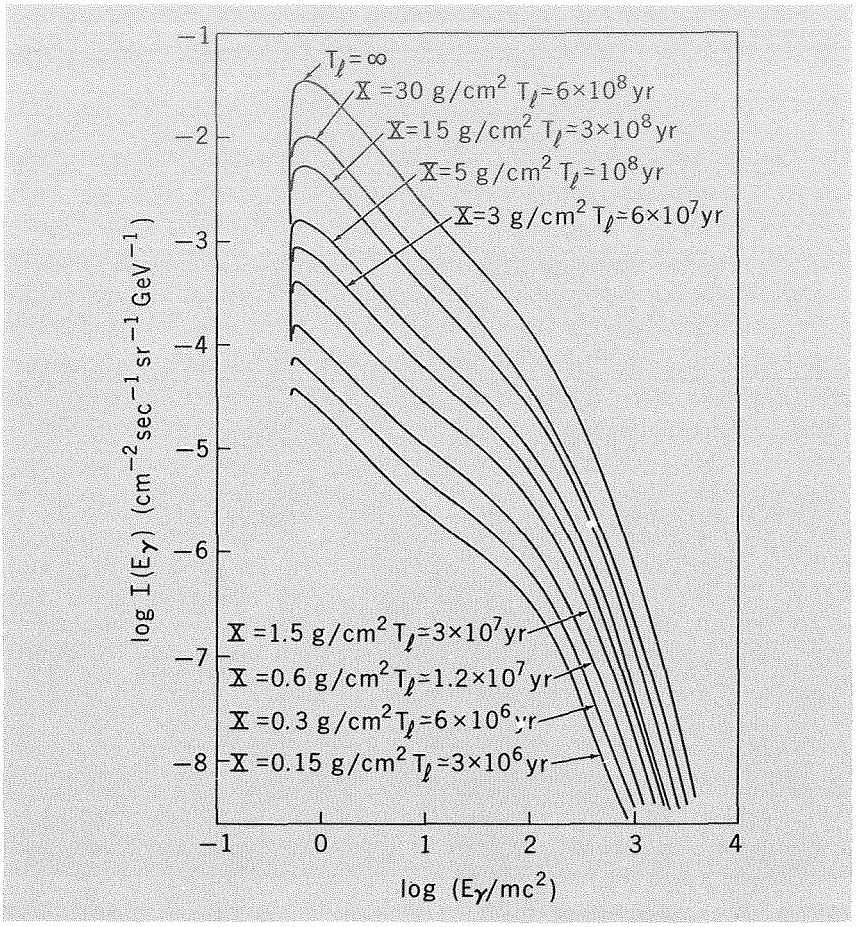

FIGURE 7–3. — The annihilation γ-ray flux spectra for the positron equilibrium fluxes given in figure 7–2 that result from annihilations in flight of positrons with energies greater than 5 keV (Stecker, 1969a).

spectra lie in the region $1/2 < \eta < 1$. This effect is due to a pileup of those γ-rays from the annihilation of relativistic positrons that are emitted in the backward direction in the cms. (See ch. 2.)

Using a value of 4 g/cm^2 for X and the formulas given in this chapter, Ramaty et al. (1970) calculated the total positron source spectrum and equilibrium flux in the galaxy from both pion-decay positrons and β-decay positrons. These spectra were calculated for various values of R_r and R_0 in equation (7–18) in order to convert from the cosmic-ray spectrum measured at the Earth to various possible galactic cosmic-ray spectra. As an extreme case, Ramaty et al. (1970) also considered power-law cosmic-ray spectra in kinetic energy per nucleon with an exponent of

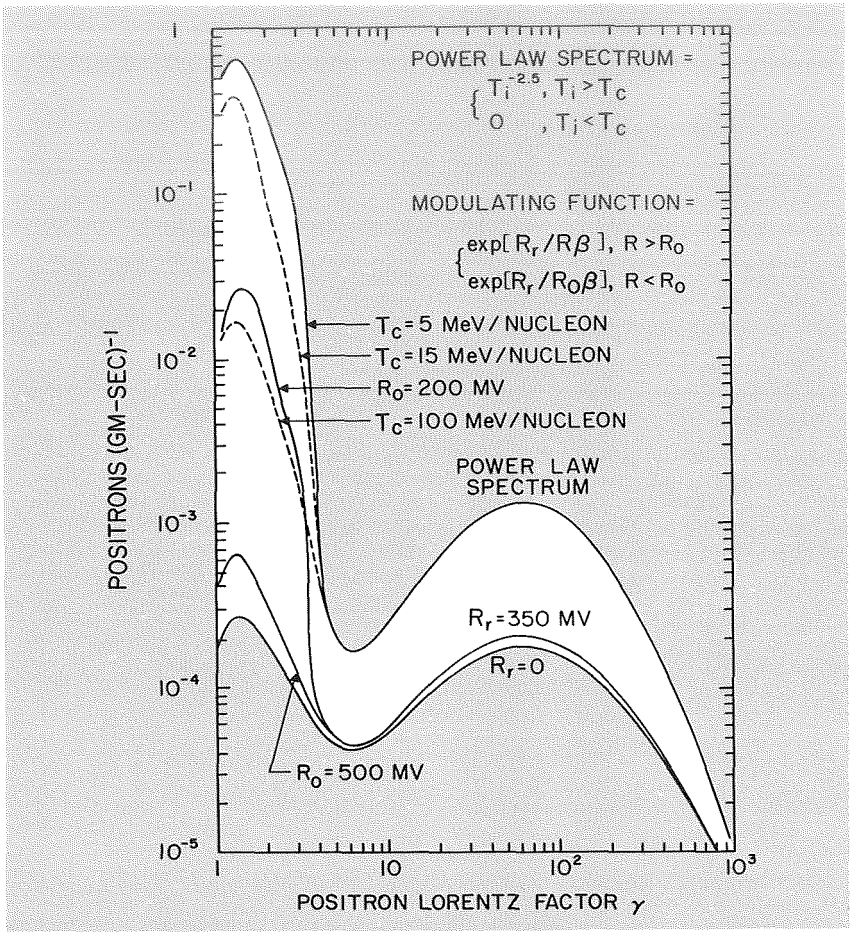

FIGURE 7–4.—Positron production spectra per gram of interstellar material calculated using a power-law cosmic-ray spectrum with various low-energy cutoffs, T_c, and demodulated solar minimum spectra with various values of R_r and R_0 (Ramaty et al., 1970).

-2.5 and various low-energy cutoffs, T_c. The source spectra calculated are shown in figure 7–4 and the resultant equilibrium spectra are shown in figure 7–5. As can be seen, the source spectra and equilibrium spectra of low-energy β-ray positrons are particularly sensitive to the various modulation parameters which determine the low-energy cosmic-ray flux. This is, of course, due to their low production threshold energy.

As can be seen, positrons with energies greater than about 2 MeV come principally from pion decay. At lower energies, the relative contribution of the β-emitters depends critically either on the value of R_0 (for $R_r = 350$ MV) or on the assumed low-energy cutoff (for a power-law spectrum).

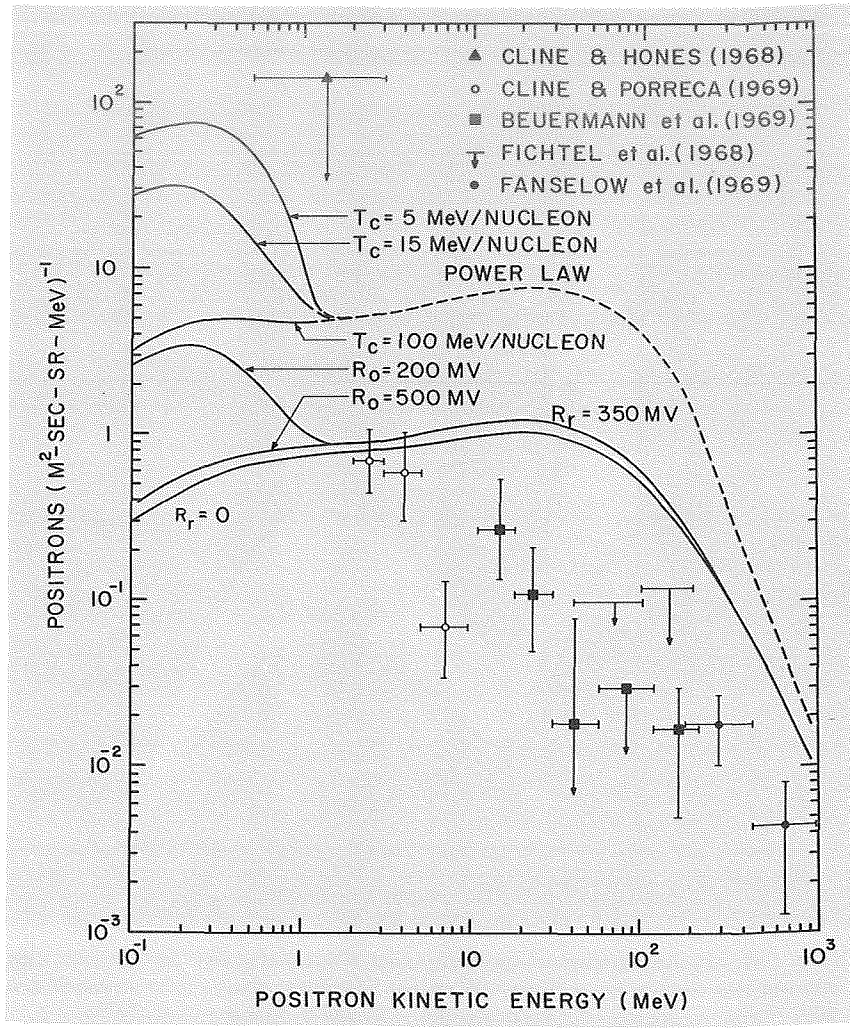

FIGURE 7–5.—Positron intensities in interstellar space together with the available positron measurements below 1 GeV (Ramaty et al., 1970).

TABLE 7–3.—*Cosmic-Ray Energy Density* (eV/cm³)

Solar minimum	$R_r = 350$ MV ($T \geq 5$ MeV/ Nucleon)		Power law		
	$R_0 = 500$ MV	$R_0 = 200$ MV	$T_c = 100$ MeV/ Nucleon	$T_c = 15$ MeV/ Nucleon	$T_c = 5$ MeV/ Nucleon
0.5.....	0.6	57	3.1	17	50

The energy densities of cosmic rays for the various spectra just discussed are given in table 7–3. As can be seen from figure 7–5 and table 7–3, unless the energy density in low-energy cosmic rays is of the order of a few tens of electron volts per cubic centimeter, most of the positron flux at ~ 1 MeV would come from pion decay rather than CNO β-decay and would be small compared to the upper limit given by Cline and Hones (1968). If spread uniformly over the galactic disk, such energy densities lead to serious difficulties regarding the stability of the galaxy (Parker, 1966). On the other hand, since the ranges of both low-energy protons and positrons are short (~ 0.1 g/cm^2), the ~ 1-MeV positron measurements sample only a small region of space which may not necessarily be representative of the galaxy as a whole.

For example, consider a 10-MeV proton; its range in hydrogen is 0.05 g/cm^2, which, for an ambient density of n atoms/cm^3, leads to a lifetime $t \approx 2 \times 10^5/n$ years. The net streaming distance corresponding to t is $a \approx (4/3\lambda\beta ct)^{1/2}$, where λ is the mean free path for diffusion. If $\lambda = 1$ pc, $a \approx 100/n^{1/2}$ pc. Since the rate of supernova explosions in the galaxy (volume $\approx 2 \times 10^{66}$ cm^3) is about 10^{-2} per year, in a spherical volume of radius $100/n^{1/2}$ pc a supernova would be expected to occur once every $2 \times 10^6 n^{3/2}$ years. For $n \gtrsim 1$ atom/cm^3, this time interval is much larger than the lifetime against ionization given before. Therefore, if these protons are produced in supernova explosions and not by more frequent events such as novae or flare stars, their intensities would exhibit sharp maxima close to the time of the explosion and decay to much lower values later on. According to Gold (1969), a supernova explosion may liberate as much as 10^{52} ergs. If 50 percent of this is in low-energy cosmic rays, the mean energy density for about $2 \times 10^5/n$ years over a sphere of radius $100/n^{1/2}$ pc would be $\sim 30\ n^{3/2}$ eV/cm^3. This is sufficient to produce a detectable flux of β-ray positrons. On a galactic scale, however, the same sources of low-energy cosmic rays occurring at a frequency of 10^{-2} per year would only produce an average energy density of $3/n$ eV/cm^3. For $n = 1$, this is somewhat large but not inconsistent with the overall energetics of the galaxy.

Since the large fluxes of low-energy cosmic rays are restricted to small volumes and short times, they do not conflict with observations that integrate over large distances, such as the temperature of H-I clouds, based on 21-cm observations, and the density of free electrons, based on the observed spectrum of nonthermal radio emission. (The connection of these quantities with low-energy cosmic rays was discussed by Balasubrahmanyan et al. (1968). The ~ 1-MeV positron measurements, however, by sampling only a small region of space corresponding to the range of the positrons of 0.2 g/cm^2 ($t \approx 10^5/n$ years, $a \approx 70/n^{1/2}$ pc), may provide evidence for local enhancement of low-energy cosmic-ray fluxes.

As can be seen from figure 7–5, given a reasonable solar modulation, the positron measurements above a few MeV are all consistent with the calculated spectra from the pion-decay process. Since the range of protons above pion production threshold is larger than 4 g/cm^2, these spectra represent mean values over time periods comparable to the positron leakage lifetime from the galaxy. The spatial and temporal inhomogeneities discussed previously, which would allow large but localized low-energy cosmic-ray fluxes, do not apply to this energy domain. The positron spectrum above a few MeV, obtained from a demodulated cosmic-ray distribution with $R_r = 350$ MV, would therefore be a good representation of the interstellar positron intensity.

7–5 THE NUMBER OF COSMIC-RAY POSITRONS ANNIHILATING NEAR REST

We now come to the problem of determining the AGS from cosmic-ray positrons annihilating near rest. Because some aspects of this determination seem deceptively simple, there has been a tendency to oversimplify this problem in the literature. For this reason, I will first list various aspects of the problem essential to an accurate analysis before proceeding to treat them.

(1) The most important source of the cosmic-ray positrons having energies greater than a few MeV is the decay of secondary charged pions. There are, however, sources of relatively low energy positrons (less than a few MeV) which, as we have seen, may have a greater probability of being trapped inside the galaxy until they annihilate near rest. These sources are the β-emitters, which are produced predominantly by low-energy cosmic-ray interactions involving carbon, nitrogen, and oxygen. Therefore, any observable 0.5-MeV line radiation from the galaxy may be primarily due to these β-emitters and an observation of the intensity of the 0.5-MeV line could supply information on the intensity of low-energy cosmic radiation in the galaxy.

(2) In considering annihilations near rest, one must consider the possibility of the intermediate formation of the bound electron-positron system; i.e., the positronium atom. At low energies, the cross section for positronium formation becomes much greater than the cross section for free annihilation.

(3) Once formed in interstellar space, a positronium atom will annihilate into three photons 75 percent of the time. This situation, as we have seen in chapter 2, contrasts sharply with the case of free annihilations, where three-photon annihilations occur with a prob-

ability of less than 1/2 percent. Therefore, the three-photon annihilation process, which produces a continuous spectrum from 0 to 0.5 MeV, must be considered along with the two-photon line annihilations. Their relative importance depends directly on the fraction of positrons which ultimately form positronium.

The positrons which annihilate near rest most likely come from two sources:

(1) Positrons from the decay of secondary charged pions which were created at low enough energies to be trapped for a sufficiently long time in the galaxy to lose essentially all their energy before either annihilating in flight or escaping from the galaxy.
(2) Positrons from the decay of β-emitting nuclei formed in collisions of low-energy cosmic rays involving nuclei of carbon, nitrogen, and oxygen.

The fraction of the original positron flux from the decay of secondary pions which annihilate near rest, f_+, is given by

$$f_+ = \frac{\int_1^\infty d\gamma\, q_+(\gamma)\, \exp\left[-\int_1^\infty \frac{d\gamma'}{|r(\gamma)|\tau_s(\gamma')}\right]}{\int_1^\infty d\gamma\, q_+(\gamma)} \tag{7-43}$$

This fraction was calculated numerically (Stecker, 1969a) using equations (7–32) through (7–38) and is given in table 7–4 and figure 7–6 for various possible mean leakage times T_l. Table 7–4 also gives the corresponding values of $Q_{\text{rest},\pi}$, the total number of positrons from pion decay per gram per second annihilating below 5 keV for the halo model of table 7–2 $Q_{\text{rest},\pi}$ being defined by the numerator of equation (7–43).

TABLE 7–4.—*Annihilations Near Rest* ($T_+ \leq 5$ *KeV*)

T_l, 10^6 yr	f_+, %	$Q_{\text{rest},\pi}$ g^{-1}-s^{-1}
∞	80	4.8×10^{-2}
600	20	1.2×10^{-2}
300	9.3	5.6×10^{-3}
100	1.6	9.6×10^{-4}
60	.61	3.7×10^{-4}
30	.14	8.4×10^{-5}
10	.022	1.3×10^{-5}
6	.0065	3.9×10^{-6}
3	.0025	1.5×10^{-6}

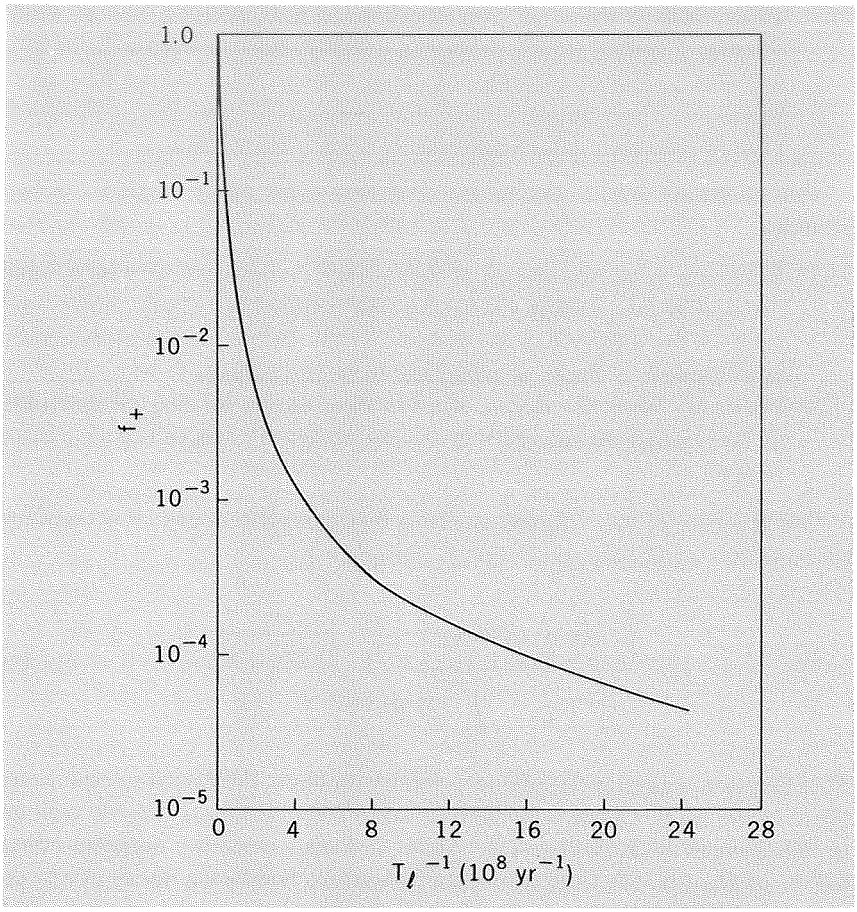

FIGURE 7–6. – The fraction of pion-produced positrons which annihilate at energies less than 5 keV as a function of mean leakage time (Stecker, 1969a).

In the case of an infinite leakage time (all positrons being trapped and annihilating in the galaxy), we find that 80 percent of the positrons produced annihilate near rest, a figure which is in perfect agreement with that given by Heitler (1960) as an asymptotic value for the annihilation of ultrarelativistic positrons when the dominant energy loss comes from ionization. However, for the leakage time usually considered as plausible for the galactic halo (10^8 years, corresponding to $X \simeq 5$ g/cm^2), only 1 to 2 percent of these positrons annihilate near rest.

Ramaty et al. (1970) made use of equation (7–43) to calculate the total number of positrons that annihilate near rest, including the contribution of β-ray positrons as well as pion-decay positrons. They used the various source spectra given in figure 7–4 for $q_+(\gamma)$. Their results for

TABLE 7–5. — *Total Positron Annihilation Rates* $Q_T(g^{-1}\text{-}s^{-1})$

Solar minimum	$R_r = 350$ MV		Power law		$I_+ = 2\times10^{-2}$ $cm^{-2}\text{-}s^{-1}\text{-}sr^{-1}$	
	$R_0 = 500$ MV	$R_0 = 200$ MV	$T_c = 100$ MeV/ Nucleon	$T_c = 5$ MeV/ Nucleon	Mean	Maximum
1.1×10^{-3}	1.8×10^{-3}	4.3×10^{-2}	3.3×10^{-2}	1.3×10^{-1}	$\frac{2.2\times10^{-2}}{n^{5/2}}$	0.5

the various assumed low-energy cosmic-ray spectra and for $X = 4$ g/cm are given in table 7–5. As an extreme case, they computed the annihilation rates that would result from an interstellar positron flux of the same order as the upper limit given by Cline and Hones (1968).

Using a total positron intensity of 2×10^{-2} particles of 1 MeV/cm²-s-sr, they also computed a maximum annihilation rate corresponding to a positron lifetime against ionization of $10^5/n$ years and a mean rate corresponding to an average time between supernova explosions of $5\times10^6 n^{3/2}$ years in a spherical volume of radius $70/n^{1/2}$ pc as discussed in section 7–4. These rates are also given in table 7–5.

7–6 GALACTIC POSITRONIUM FORMATION

The cross section for positronium formation by fast positrons in atomic hydrogen has been calculated by Cheshire (1964) and will be used as an approximation for the interstellar medium. The ratio of positronium formation to free annihilation is only significant at nonrelativistic energies and may be approximated by

$$\frac{\sigma_{pos}(\gamma)}{\sigma_A(\gamma)} \equiv S(\gamma) = \begin{cases} 0 & (\gamma-1) > 10^{-2} \\ 10^{-6}(\gamma-1)^{-3} & 10^{-4} < (\gamma-1) < 10^{-2} \\ 10^{-2}(\gamma-1)^{-2} & \text{for } 2.5\times10^{-5} < (\gamma-1) < 10^{-4} \\ 6.4\times10^{11}(\gamma-1) & 10^{-5} < (\gamma-1) < 2.5\times10^{-5} \\ 0 & (\gamma-1) < 10^{-5} \end{cases} \tag{7–44}$$

At energies of the order of the hydrogen binding energy, the probability for positronium formation in the ground state has been estimated to be between 0.25 and 0.50 and the probability for positronium formation in excited states is small (Deutsch, 1953).

Equation (7–44) was used by Stecker (1969a) to determine the amount of positronium being formed by positrons combining with electrons as a function of energy. The result of this calculation is shown in figure 7–7.

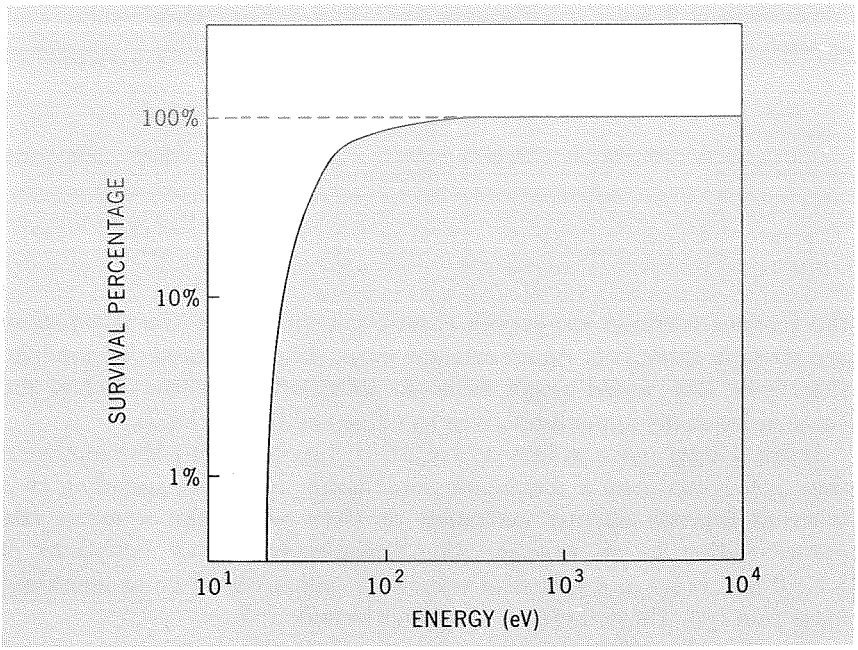

FIGURE 7–7. – The percentage of positrons which, having survived to reach an energy of 10 keV, survive to reach lower energies. The dashed line indicates the survival fraction found when only free annihilations are considered; the solid line indicates the survival fraction when positronium formation is also taken into account (Stecker, 1969a).

This figure shows the percentage of positrons which, having survived to reach a 10-keV kinetic energy, survive to reach lower energies. The dashed line indicates the survival fraction for free annihilation only; the solid line indicates the survival fraction when positronium formation is taken into account. Figure 7–7 shows that almost all of the positrons annihilating near rest do so through intermediate positronium formation with an average energy of about 35 eV.

7–7 THE INTENSITY OF THE POSITRON ANNIHILATION LINE AT 0.51 MeV

It follows from the results of the last section and from the discussion in chapter 2 that galactic cosmic-ray positrons annihilate primarily from an S state of positronium with 75 percent of these annihilations producing a three-photon continuum rather than a two-photon line at 0.51 MeV. Therefore, on the average, one 0.51-MeV photon is produced for every two positrons which annihilate. Thus, the intensity of 0.51-MeV γ-rays

observed along the line of sight as a function of galactic coordinates is given by

$$I_{0.51}(l^{II}, b^{II}) = \frac{Q_{T,\,\mathrm{rest}} M(l^{II}, b^{II})}{8\pi} \tag{7-45}$$

where $M(l^{II}, b^{II})$ in g/cm^2 is the amount of interstellar gas in the direction of observation.[2] Equation (7–45) was used by Ramaty et al. (1970) to determine possible intensities of the 0.51-MeV annihilation line in different directions in the galaxy based on the values of $Q_{T,\,\mathrm{rest}}$ given in table 7–5. The results of this calculation and their implications will be given in the next chapter, where we will complete our discussion of galactic γ-ray production.

[2] The symbols l^{II} and b^{II} denote galactic longitude and latitude respectively.

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